VIBRATION TESTING CONTROL USING SPECTRAL WARping

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VIBRATION TESTING CONTROL SYSTEMS: GENERAL OBJECTIVES

Testing is one of the most important steps in creating reliable and durable prototypes of high-performance machinery. It is also crucially important in the operational development of fine engineering prototypes. The objectives of testing are to provide for complete simulation of all possible stresses on a product in service and to analyse the product's behaviour and performance under service conditions. Essentially the stress factors are linear and angular acceleration, impacts, vibration, acoustic environment and climate.

Testing for robustness and reliability under vibration is gaining in importance for engineering systems such as aircraft, motor-vehicles, power generating equipment, automatic systems, computers, radio, electronic and data-conditioning equipment and so on. The background to this is that approximately 70% of all in-service failures in the engineering industry result from vibration: it initiates stresses that exceed the design strength, causes endurance failures, reduces the operational life of the product, ensures safety for personnel, saves fuel and lubricants, and allows fast variation of the vibration modes. In some cases laboratory testing is practically the only feasible method (for example in rocketry). It reduces the amount of field testing required and thereby lowers its cost.

Real vibrations are usually wideband random processes described by their probability and statistical characteristics. Their use allows us to analyse the dynamical properties and the control of an object in a quasi-static fashion. In such cases the vibrational state of the object is commonly defined by the power spectral characteristics of random movements or accelerations at definite points of the object under test. Further discussion can be found in Yarmolik and Demidenko (1).

The requirements placed upon the accuracy and speed of large-scale experimental data processing during vibration testing are so stringent that they become impossible without application of special computer-based control systems: the design of such systems has therefore become very important.

In the general case, the primary task of the control system may be formulated as follows. Let \( G' \) be the power spectrum characterizing the required vibration state at some specified point of an object under test. We assume that \( G' \) is discrete, consisting of \( m \) distinct spectral components of which \( g_j(k) \) is the \( k \)-th: \( G' = (g_1(0), g_1(1), \ldots, g_1(m - 1)) \). It is necessary to find the spectrum \( G = (g(0), g(1), \ldots, g(m - 1)) \) of \( X \) the random process input to the vibration equipment, then synthesise this process and apply it to the vibration equipment so as to cause the output process \( Y \) to have a power spectrum \( G \), differing from \( G' \) by an error \( E = (e(0), e(1), \ldots, e(m - 1)) \) whose components are lower than those of a specified set of values \( B = (b(0), b(1), \ldots, b(m - 1)) \), i.e. to satisfy the set of \( m \) conditions

\[
e(k) = |g_j(k) - g_j(k)| < b(k) \quad \ldots (1)
\]

The input process thus obtained can then be applied to the object for a specified time interval to maintain the specified vibration state at the object output.

To attain these ends, a series of problems has to be solved. The major ones are:

(a) spectral analysis of the output process \( Y \) to define the spectral estimate \( G \);
(b) generation of the realisation of the input process \( X \) with specified power spectrum \( G \);
(c) identification of the transfer function \( H \) of the system from input \( X \) to output \( Y \);
(d) manipulation of the input signal \( X \) (and spectrum \( G \)) in accordance with error \( E \) obtained.

The solution of the first two problems, i.e. generation and analysis of random processes, is usually based on the discrete Fourier transform (DFT) computed by means of the basic fast Fourier transform (FFT) of Cooley and Tukey (2), or more recent versions of the algorithm. In many cases the testing task demands the achievement of the specified output process spectrum with non-uniform frequency resolution (e.g. with logarithmic resolution). In such cases, the direct application of the DFT with its uniform frequency resolution (determined by the finest resolution required) leads to a sharp increase in the size of the resulting arrays and certainly to a decrease in the performance of the control system.

The special digital signal processing technique, spectral warping, proposed by Oppenheim et. al. (3) in and elaborated in Oppenheim and Johnson (4), offers the possibility of increasing the effectiveness of the DFT used in implementing such tasks. This technique consists of transforming the original sequence to a new one having the property that equally-spaced frequency samples of its DFT are identical to unequally-spaced frequency samples of the DFT of the original sequence. This opens up the possibility of decreasing the array size of the DFT and thereby increasing its performance, as measured, for example, by its speed of execution. In this paper we propose a novel general organisational structure and functional algorithm for a vibration testing control system, which makes use of this spectral warping approach.

AMPLITUDE AND PHASE CHARACTERISTICS OF THE SPECTRAL WARping NETWORK

In order to apply the spectral warping network correctly in an automation control system we need to understand the characteristics of all the signal distortions which this network introduces.

The spectral warping network described in (3) and (4), and characterised as an all-pass digital filtering cascade is shown in Figure 1.

This network produces the signal frequency distortion which can be described by the analytical expression as follows

...
where \( \omega \) - angular frequency of the network input signal; \( \omega' \) - angular frequency of the network output signal; \( T \) - sampling interval.

Figure 2 shows the frequency-warping function for several values of the warp parameter \( a \).

Let us now examine the amplitude-frequency and phase-frequency characteristics of such a cascade, which play an important role in applying the spectral warping network.

The system function corresponding to the multi-section spectral warping network is given by

\[
N(z) = \prod_{i=1}^{N} (1 - a) + \frac{1}{1 - e^{j\omega T}}
\]

Taking the first section alone and setting \( z^{-1} = \cos \omega T - j \sin \omega T \), we obtain

\[
H_0(z) = \frac{1}{1 - az^{-1}}
\]

so that the amplitude and phase characteristics are

\[
|H_0(z)| = 1 - 2a \cos \omega T + a^2
\]

Similarly, taking the first two sections together we find, from their composite system function

\[
H_2(z) = \frac{(1 - a) + \frac{1}{1 - e^{j\omega T}}}{1 - e^{j\omega T}}
\]

after some algebraic reduction, that

\[
H_2(z) = \frac{(1 - a) + \frac{1}{1 - e^{j\omega T}}}{1 - e^{j\omega T}}
\]

with corresponding amplitude and phase characteristics

\[
|H_2(z)| = 1 - 2a \cos \omega T + a^2
\]

Thus we see that the effect of the frequency characteristic can be substantial. For example, the amplitude shows a ripple with period equal to the sampling frequency; Figure 3 shows the ratio of the amplitudes at dc and the half-sampling frequency \( \frac{1}{2} \). Further details may be found in Demidenko and Lever (5).

Thus the frequency characteristics should be certainly kept in mind when implementing the spectral warping network in automation control systems. Let us consider now the implementation of a spectral warping network in a control system oriented towards vibration testing.

AUTOMATED VIBRATION TESTING CONTROL SYSTEM

Figure 4 shows the general form of the Automated Vibration Testing Control System proposed. The functional operation of the consists of a set of related modes, namely:

(a) definition of warping parameter value, \( a \);
(b) identification mode;
(c) zero-order approximation calculation mode;
(d) control mode;
(e) vibration testing mode.

During the execution of mode (a) it is necessary with regard to the specified spectrum \( G' \) (with, in general, a non-uniform frequency resolution) to find the value of the warping parameter \( a \) which will provide the required spectral transformation from non-uniform to uniform type, in order to provide spectral analysis with a DFT of reduced size. This can be done using diagrams of the type shown in Figure 2, or by calculations based on the analytical expression in Equation (2). This value of the parameter \( a \) specifies the spectral warping transformation performed by the second spectral warping network (SWNW). At the same time the inverse value \( -a \) is employed in the first spectral warping network specifying the signal spectrum transformation from the uniform frequency type to the required non-uniform one.

During mode (b) the white noise generator (WNG) forms, for a specified r. m. s. level \( \sigma_0 \), a white noise random process \( X_n \) which is applied through the first spectral warping network.
(SWN1) and a digital-to-analogue converter (DAC) to the vibration bench on which the object under test is mounted. At the same time the Fourier transform of the process \( x_{an}(k) \) is calculated by means of a spectral analysis algorithm according to the DFT expression

\[
s_{an}(k) = \frac{1}{N} \sum_{n=0}^{N-1} x_{an}(n) \exp(-j2\pi kn/N) \quad \ldots \quad (13)
\]

where \( s_{an}(k) \) - \( k \)-th coefficient of the Fourier transform \( S_{an} \) of the realisation of process \( X_{an} \),

\( x_{an}(n) \) - \( n \)-th sample of the realisation of process \( X_{an} \),

\( N \) - number of samples in the segment of the realisation of process \( X_{an} \).

Using \( S_{an} = (s_{an}(0), s_{an}(1), \ldots s_{an}(m-1)) \) one can obtain the power spectrum samples \( G_{an} = (g_{an}(0), g_{an}(1), \ldots g_{an}(m - 1)) \):

\[
g_{an}(k) = \frac{s_{an}(k) s_{an}^{*}(k)}{\sigma_{n}^{2}} \quad \ldots \quad (14)
\]

where \( s_{an}(k) \) is the complex conjugate of \( s_{an}(k) \). Then the set of Fourier coefficients \( S_{an} \) and the power spectrum \( G_{an} \) of the output process \( Y_{an} \) are defined. It is customary to use time- and/or frequency windowing to improve the quality of the estimates of the characteristics of \( X_{an} \) and \( Y \).

Finally, in mode (b), the transfer function for the chain: [first spectral warping network - digital-to-analogue converter - vibration bench - object under test - transducer - analogue-to-digital converter - second spectral warping network] is defined as \( H = (h(0), h(1), \ldots, h(m - 1)) \), by use of the expression

\[
h(k) = \frac{s(k) s^{*}(k)}{\sigma_{h}^{2}} \quad \ldots \quad (15)
\]

where \( \sigma_{h}^{2} \) is the power of the white noise input process.

Note that the main requirement for the white noise generator used in mode (b) is that it has a continuous uniform power spectrum and a Gaussian amplitude probability distribution. The generators described in Coates et al. (6) and Tsor et al. (7) are suitable for this purpose.

The task of the zero-order approximation calculation operation (mode(c)) is to find the first spectral approximation of the input test signal whose effect on the object tested will produce the output signal with required spectrum \( G_{a} \).

During the first step of the operation the reference power spectrum \( G_{a} \) for the SWN2 output signal is calculated by the use of the specified warping spectrum \( G_{w} \), and the power spectrum warping network system function expression Equation (12), bearing in mind the expression linking the output and input spectral representations:

\[
g_{swn2}(k) = |h_{swn2}(k)|^{2} g_{a}(k) \quad \ldots \quad (16)
\]

where \( h_{swn2}(k) \) is the \( k \)-th sample of the SWN2 transfer function \( H_{swn} \).

The second step of this operation consists in the calculation of the zero-order approximation as follows

\[
g_{a}(k) = \frac{g_{swn2}(k)}{|h_{w}(k)|} \quad \ldots \quad (17)
\]

where \( g_{swn2}(k) \) is the \( k \)-th sample of the zero-order approximation to the spectrum of the input process \( X \). Here and subsequently, the bracketed superscript refers to the number of times the approximation procedure has been iterated during the control mode.

The first step in the execution of the fourth (control) mode is the generation of the zero-order realisation of the random process with specified power spectrum, by means of Rice-Pierson scalar model (8)

\[
x_{a}(n) = \sum_{k=0}^{\infty} \sqrt{g_{a}(k)} \cos(2\pi kn + \theta_{k}) \quad \ldots \quad (18)
\]

where \( T \) - sampling interval;

\( \Delta_{w} \) - appropriate frequency sampling interval;

\( \theta_{k} \) - independent random phase uniformly distributed in the interval \( (0, 2\pi] \);

\( m \) - number of frequency components.

This can be accomplished by using the FFT algorithm in the random process generator (RPG), with the present phase values produced by the white noise generator (WNG).

Note that the use of Rice-Pierson decomposition for generating the drive signal in digital automation vibration testing control systems is essentially equivalent to substituting a wideband random process for a polyharmonic process. The justification for such a substitution has been investigated, for example, by Kolovski (9), where it has been shown that, for a uniform random phase distribution and a large enough number of harmonics in the generated process, the momentary responses of the latter that have been calculated (a) by averaging over the ensemble of realisation and (b) by over time coincide, for all practical purposes, with the respective Gaussian process responses.

The synthesised process is sent to the first spectral warping network which performs the appropriate spectral distortion according to the value of the warping coefficient \( \omega \). Then the signal is passed through the DAC to the vibration bench control input and produces certain vibrational disturbances to the object under test. The process obtained from a transducer placed on the object incorporates the vibrational state of the object and this process is sent through the ADC to SWN2. This network predistorts the process in such a manner that the spectral components which are required are placed at some frequencies uniformly distributed along the frequency scale. Then spectral analysis is performed with the use of the DFT algorithm.

Usually the conditions in Equation (1) will not be achieved during the initial (zero-order) iteration, due to the presence of nonlinear features in the vibration bench-object system, and due to inaccuracies of calculation stemming from computer word length limitation. In this case one needs to use some an iterative algorithm to improve the input process in order to satisfy the condition Equation (1). The stochastic approximation algorithm is one such, and can be described as follows

\[
g_{a}(k) = g_{a}^{(0)}(k) + \gamma \frac{|g_{swn2}(k) - g_{swn2}(k)|}{|h_{w}(k)|} \quad \ldots \quad (19)
\]

where \( \gamma \) is the convergence factor during the \( i \)-th iteration.

After delegation of each new approximation to the input signal spectrum the whole cycle of the control mode is repeated until the condition Equation (1) is satisfied. When the condition is satisfied the automatic vibration testing control system records the resulting input process in the computer memory and moves on to the vibration testing mode.

While the testing mode is being executed the stored input process realisation is applied through SWN1 and DAC to the vibration bench control input for a specified time. During this time, output processes are analysed and condition Equation (1) is tested at regular intervals. When needed, input processes can be corrected by using the control mode.
CONCLUSIONS

Previous investigations in spectral prewarping originally introduced in (3) and (4) were conducted mainly in the context of spectral analysis with high resolution in selected frequency regions, oriented towards the particular task of speech correction. This paper presents some new analytical results which describe the frequency characteristics of the spectral warping network and which occupy an important place in the practical implementation of this technique. We have shown how these results can provide the basis for creating a new structure for an automated vibration testing control system using spectral prewarping networks. This automatic vibration testing control system structure can provide improvements in the frequency resolution of vibration testing equipment without additional requirements on the performance of the computer used.

It is important to mention that in numerous cases in real working conditions the object has vibrational motions in more than one space dimension. Hence laboratory simulation of actual vibrations requires at least three force exciters arranged in orthogonal directions and a correspondingly modified implementation of the control system. Such a system must be able to generate the three-dimensional random process with specified spectra in each dimension and specified cross-spectra modelling the coupling between dimensional components. The system must also be able to analyse such processes. This requires the solution of the problems of designing and implementing a three-dimensional spectral warping network. We strongly believe that the solution will be applicable not only to control systems but also to other numerous practical signal processing areas, for example, in image processing and recognition, and we suggest these tasks for future investigation.

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REFERENCES


Figure 1: Spectral-warping network
Figure 2: Frequency-warping transformation for 
\( a = 0.5, 0.25, 0.0, -0.25 \) and -0.5 
(frequencies normalised by the sampling frequency)

Figure 3: Attenuation at the half-sampling frequency, relative 
to unity dc gain
Figure 4: Automation vibration testing control system structure

SFD - system function definition;  
C - control; RPG - random process generation;  
SWN1, SWN2 - 1st and 2nd spectral warping network resp.;  
DAC - digital-to-analogue conversion; VB - vibration bench;  
O - object under test; T - transducer;  
ADC - analogue-to-digital conversion;  
SA - spectral analysis; WNG - white noise generation